Semi-Log Model

- The slope coefficient measures the relative change in Y for a given absolute change in the value of the explanatory variable (t).

Using calculus:

$$b_1 = \frac{\partial \ln Y}{\partial t} = \frac{\ln Y}{Y} \frac{\partial Y}{\partial t} = \frac{\partial Y}{\partial t} \cdot \frac{1}{Y}$$

If we multiply the relative change in Y by 100, we get the percentage change or growth rate in Y for an absolute change in t.
Log (GDP) 1969-83

- Log(Real GDP) = 6.9636 + 0.0269t
- se (.0151) (.0017)
- R² = .95
  - GDP grew at the rate of .0269 per year, or at 2.69 percent per year.
  - Take the antilog of 6.9636 to show that at the beginning of 1969 the estimated real GDP was about 1057 billions of dollars, i.e. at t = 0

Compound Rate of Growth

- The slope coefficient measures the instantaneous rate of growth
- How do we get r -- the compound growth rate?
  - b₂ = ln (1 + r)
  - antilog (b₂) = (1 + r)
  - So r = antilog (b₂) - 1
  - So r = antilog(.0269) -1
    - r = 1.0273 - 1 = .0273
Linear-Trend Model

- Trend model regresses $Y$ on time.
  - $Y_t = b_1 + b_2t + e_t$
  - This model shows whether GNP is increasing or decreasing over time.
  - The model does not give the rate of growth.
  - If $b_2 > 0$, then an upward trend.
  - If $b_2 < 0$, then a downward trend.

Linear-Trend Example

- $\text{GNP} = 1040.11 + 34.998t$
- $\text{se} (18.85) (2.07) \ R^2 = .95$
  - GNP is increasing at the absolute amount of $35$ billion per year.
  - There is a statistically significant upward trend.
- Growth model measures relative performance
- Trend model measures absolute performance
3. Lin-Log Models

Lin-Log Model
- The dependent variable is linear, but the explanatory variable is in log form.
  - Used in situations for example where the rate of growth of the money supply affects GNP.
Lin-Log Example

- GNP = b_1 + b_2 \ln M + e
  - The slope coefficient is d\text{GNP}/d\ln M
    - It measures the absolute change in GNP for a relative change in M.
  - If b_2 is 2000, a unit increase in the log of the money supply increases GNP by $2000\text{ billion}.
    - Alternatively, a 1% increase in the money supply increases GNP by \frac{2000}{100} = $20\text{ billion}.

In this case, we need to divide by 100 since we are changing the money supply change from a relative change to a percentage change.
4. Functional Form Summary

Data

- GNP and money supply over the period 1973-87 in the U.S.
- GNP in billions of dollars = Y
  - Mean = 2791.47
- M2 in billions of dollars = X
  - Mean = 1755.67
Log-linear model

- \( \ln Y = 0.5531 + 0.9882 \ln X \)
- How interpret?
  - The slope coefficient is \( \frac{d\ln Y}{d\ln X} \)
    - i.e. relative change in \( Y \) / relative change in \( X \)
  - For a 1% increase in the money supply, the average value of GNP increases by .9882% (almost 1%)
Log-Lin Model

- \( \ln Y = 6.8616 + 0.00057X \)

How interpret?
- The slope coefficient is \( \frac{d\ln Y}{dX} \)
  - i.e. relative change in \( Y \) / absolute change in \( X \)
- For a billion dollars rise in the money supply, the log of GNP rises by .00057 per year.
  - To make a %, multiply by 100: GNP rises by 0.057% per year.

Log-Lin Model

- How to convert to an elasticity?
  - The slope coefficient is:
    \[
    b_z = \frac{\partial \ln Y}{\partial X} = \left( \frac{1}{Y} \right) \frac{\partial Y}{\partial X}
    \]
  - To get an elasticity multiply by \( Y \)
    .00057(175.567) = 1.0007
  - So a 1% increase in the money supply leads to a 1.0007% increase in GNP
Lin-Log Model

- \( Y = -16329.0 + 2584.8 \ln X \)

- How interpret?
  - The slope coefficient is \( \frac{dY}{d\ln X} \)
    - i.e. absolute change in \( Y/ \) relative change in \( X \)
  - A unit increase in the log of the money supply increases GNP by 2584.8 billion dollars.
    - If money supply rises by 1%, GNP rises by $26 billion dollars.

Lin-Log Model

- How to convert to an elasticity?
  - The slope coefficient is:
    \[
    b_2 = \frac{\partial Y}{\partial \ln X} = \left( \frac{X}{T} \right) \frac{\partial Y}{\partial X}
    \]
    To get an elasticity divide by \( Y \)
    \[
    \frac{2584.8}{279} = 9.260
    \]
    A 1% increase in the money supply leads to a 9.260% increase in GNP.
Linear Model

- $Y = 101.20 + 1.5323X$

How interpret?
- The slope coefficient is $dY/dX$
  - i.e., absolute change in $Y$/absolute change in $X$
- For a $1$ billion increase in the money supply increases GNP by $1.5323$ billion dollars.

Linear Model

- How to convert to an elasticity?
  - The slope coefficient is: $dY/dX$
  - Multiply this coefficient by $Xbar/Ybar$
    - $1.5323 \times (1755.67/2791.47) = .9637$
  - A 1% increase in the money supply leads to a .9637% increase in GNP
Monetarist Hypothesis

- Can test the monetarist hypothesis with double log model
  - 1% increase in money supply leads to a 1% increase in GNP
    - A t-test reveals that coefficient not different from 1.

Summary

- Models are similar:
  - Elasticities are similar.
  - R² are similar
    - Can only compare same similar dependent variables
  - All t values are significant
- Not much to choose among models.
  - Depends on issue-elasticity, growth, absolute change, etc.
5. Reciprocal Model

Reciprocal Model

- \[ Y = b_1 + b_2 \left( \frac{1}{X} \right) + e \]

- Model is linear in the parameters, but nonlinear in the variables
- As \( X \) increases,
  - The term \( 1/X \) approaches 0
  - \( Y \) approaches the limiting value of \( b_1 \).
Fixed Cost Example

- Average fixed cost of production declines continuously as output increases:
  - Fixed cost is spread over a larger and larger number of units and eventually becomes asymptotic.

Phillips Curve Example

- Sometimes the Phillips curve is expressed as a reciprocal model
  - $Y = b_1 + b_2(1/X) + e$
    - $Y =$ rate of change of money wages (inflation)
    - $X =$ unemployment rate.
Phillips Curve Example

- The curve is steeper above the natural unemployment rate than below.
  - Wages rise faster for a unit change in unemployment if the unemployment rate is below the natural rate of unemployment than if it is above.

Phillips Curve Example

- Suppose we fit this model to data.
    - $Y = -1.4282 + 8.7243 \frac{1}{X}$
    - $se = (2.068) (2.848)$
  - This shows that the wage floor is -1.43%
  - As the unemployment rate increases indefinitely, the % decrease in wages will not be more than 1.43 percent per year.
6. Polynomial Regression Models

Polynomial Model

- These are models relating to cost and production functions
- Ex: Long run average cost and output
- LRAC curve is a U-shaped curve.
  - Capture by a quadratic function (second degree polynomial):
    - LRAC = b_1 + b_2Q + b_3Q^2
Polynomial Model

- In stochastic form:
  - LRAC = b₁ + b₂Q + b₃Q² + e
- We can estimate LRAC by OLS.
- Q and Q² are correlated
  - They are not linearly correlated so do not violate the assumptions of CLRM.

S & L Example

- Use data for 86 S&Ls for 1975.
  - Output Q is measured as total assets
  - LRAC is measured as average operating expenses as % of total assets
- Results:
  - LRAC = 2.38 - 0.615Q + 0.054Q²
S & L Example

- This estimated function is U-shaped.
- Its point of minimum average cost if reached when total assets reach $569 billions:
  - \( \frac{dLAC}{dQ} = -0.615 + 2(0.054)Q \)
  - Set equal to 0
  - \(-0.615 + 0.108 Q = 0\)
  - \(Q = \frac{0.615}{0.108} = 569\)

S & L Example

- This is used by regulators to decide whether mergers are in the public interest and also by managers to decide on efficient scale.
- It turns out that most S&Ls had substantially less than $74 in assets, so mergers or growth ok.
END OF CHAPTER 6