Κεφάλαιο 8ο: Διαφορικές εξισώσεις

8.1 H entolή DSolve

Se autó to kejálaio qa ascolhqoúme me thn lúsh diajorikώn exisώsewn dhl. exisώsewn me paragώgouV stiV opoíeV eínai ágnwsth mia ή perissótereV sunartήseiV y[x] **kai pou h parágwgoV touV** y'[x] **ikanopoieí tiV exisώseiV. P.c** x^2 y'[x]-y[x]==0 (**ágnwsth h** y[x]) **kai to sústhma** y'[x]+y[x]-z[x]==Sin[x],z'[x]+y[x]+z[x]==Cos[x] **me ágnwsteV sunartήseiV tiV** y[x],z[x]. **Duo eínai oi basikéV sunartήseiV h** DSolve **kai h** NDSolve. H **deúterh maV parécei thn dunatóthta ariqmhtikήV epílushV diajorikώn exisώsewn kai susthmátwn kai crhsimopoieí proseggistikéV meqódouV enώ h prώth prospaqeí na breí mia akribή lúsh twn exisώsewn.**

```
?DSolve
```

```
DSolve@eqn, y, xD solves a differential equation for
   the function y, with independent variable x. DSolve@
   8eqn1, eqn2, ... <, 8y1, y2, ... <, xD solves a
   list of differential equations. DSolve@eqn, y, 8x1,
   x2, ... <D solves a partial differential equation. More…
```

dhl. h DSolve[eqn, y, x] **lúnei thn diajorikή exíswsh** eqn **kai brískei thn sunárthsh** y, **h opoía écei anexárthth metablhtή thn** x. **H** DSolve[{eqn1, eqn2, ... }, {y1, y2, ... }, x] **lúnei to sústhma** eqn1, eqn2, ... **apo diajorikéV exisώseiV me ágnwsteV tiV sunartήseiV** y1, y2, ... **pou eínai sunartήseiV thV** x. **TéloV h** DSolve[eqn, y, {x1, x2, ... }] **lúnei thn exíswsh merikώn paragώgwn** eqn **wV proV thn** y **pou eínai sunárthsh twn metablhtώn** x_1, x_2, ... **Akolouqoún arketá paradeígmata (gia didaktikoúV lógouV gráyame katá "láqoV"** DSolve[eqn,y[x],x] **antí tou swstoú** DSolve[eqn,y,x])

```
Remove@eqn, y, xD
eqn = x^2 y'@xD − y@xD « 0;
DSolve@eqn, y@xD, xD

88y@xD → Ą^(−1êx) C@1D<<
```

Blépoume óti h genikή lúsh periécei mia staqerá C[1]. **An perilambánontai kai álleV staqeréV sthn lúsh autéV qa apariqmoúntai wV** C[1],[2],....**Mporoúme na alláxoume ton sumbolismó twn staqerώn ópwV maV arései p.c to** C **me** sta8era **me thn entolή** DSolveConstants->sta8era **p.c**

```
soll = DSolve@eqn, y@xD, x, DSolveConstants → sta8eraD

88y@xD → Ą^-1êx sta8era@1D<<
```

Prosέ**xte** ό**ti h l**ύ**sh** y[x] **den d**ί**netai apeuqe**ί**aV all**ά **m**έ**sa se** {{}}!!! **dhl**. **m**έ**sa se d**ύ**o l**ί**steV** ό**pwV ja**ί**netai kai apo thn** FullForm:

```
FullForm@sollD

List@List@Rule@y@xD,
   Times@Power@E, Times@-1, Power@x, −1DDD, sta8era@1DDDDD
```

PwV ό**mwV qa mpor**έ**some na "xetrup**ώ**soume" thn** y[x]] **apo kei m**έ**sa gia na bro**ύ**me gia par**ά**deigma to tetr**ά**gwn**ό **thV**; **Qa anaj**έ**roume d**ύ**o tr**ό**pouV**:
Prώ**toV tr**ό**poV crhsimopoi**ώ**ntaV thn** Last ή **isod**ύ**nama pa**ί**rnontaV to stoice**ί**o** 2o **thV** soll[[1,1]] **dhl**. **to** soll[[1,1]][[2]] **pou gr**ά**jetai aplo**ύ**stera** soll[[1,1,2]]

```
soll@@1, 1, 2DD
Last@soll@@1, 1DDD

Ą^-1êx sta8era@1D
```

```
Ą^-1êx sta8era@1D
```

```
Ą^-1êx sta8era@1D
```

Deύ**teroV tr**ό**poV**: **crhsimopoi**ώ**ntaV t**o **s**ύ**mbolo** /. **thV antikat**ά**stashV**:

> *expr* /. *rules* **ejarm**ό**zei touV kan**ό**neV pou br**ί**skontai sta** rules **gia na metaschmat**ί**sei k**ά**qe tm**ή**ma thV** *expr*.

```
y@xD ê. soll

8Ą^-1êx sta8era@1D<
```

Epeidή ό**mwV up**ά**rcoun p**ά**li ta** {} **qa pr**έ**pei kal**ύ**tera na gr**ά**youme**

```
y@xD ê. soll@@1, 1DD

Ą^-1êx sta8era@1D
```

ή isodύnama

```
y@xD ê. soll@@1DD

Ą^-1êx sta8era@1D
```

(To y[x]/.soll[[1]] aplά shmaίnei όti brίskoume prώta to y[x]/.soll dhl. th lίsta 8Ą^-1êx sta8era@1D< kai metά brίskoume to 1o stoiceίo thV lίstaV dhl. to Ą^-1êx sta8era@1D(=y[x]/.soll[[1]]).)
Tώra apo thn stigmή pou έcoume ton tύpo thV y[x] mporoύme na kάnoume dokimή an dhl. prάgmati ikanopoieίtai h arcikή maV diajorikή exίswsh:

```
lysh = soll@@1, 1DD
eqn ê. lysh

y@xD → Ą^-1êx sta8era@1D
```

```
-Ą^-1êx sta8era@1D + x^2 y'@xD == 0
```

DustucώV den apέdwse h antikatάstash y@xD → Ą^-1êx sta8era@1Dmέsa sthn eqn(den pήrame True) .Autό sumbaίnei diόti gίnetai antikatάstash thV sunάrthshV y[x] allά όci kai thV paragώgou thV y'[x] mέsa sthn eqn!!! Me y@xD → Ą^-1êx sta8era@1Dennooύme antikatάstash tou y[x] me Ą^-1êx sta8era@1Dkai tίpota parapάnw. Me authn thn antikatάstash den mporeί na katalάbei to Mathematica όti me y[x] ennooύme sunάrthsh tou x !!! Opόte den έcei kammiά idέa gia to pwV qa breί mia timή thV y ή pwV qa paragwgίsei thn y. To mόno pou xέrei eίnai h timή thV y[ ] όtan έcoume qέsei to grάmma x. Opόte qa prέpei prώta na kάnoume thn antikάtάstash thV y[x]me y[x]/.soll[[1,1]] kai tou x me 3, gia na upologίsoume gia parάdeigma to y[3] kai prέpei na broύme thn parάgwgo y1 thV y kai metά na antikatastήsoume sthn eqn sugcrόnwV tiV antikatastrάtieV twn y[x],y'[x] gia na kάnoume thn dokimή. Thn parάgwgo (se morjή kanόna) thn brίskoume me D[soll,x][[1,1]] kai se morjή sunάrthshV me D[y[x]/.soll[[1,1]],x]. AV doύme ta parakάtw apotelέsmata

```
y@3D = y@xD ê. soll@@1DD ê. x → 3
paragwgoslyshs = D@soll, xD@@1, 1DD
y1@xD = D@Hy@xD ê. soll@@1, 1DDL, xD
eqn ê. 8lysh, y '@xD → y1@xD<
eqn ê. 8lysh, paragwgoslyshs<

sta8era@1D
 Ą1ês
```

```
yÅ@xD →  Ą−1êx sta8era@1D
          x²
```

```
Ą−1êx sta8era@1D
     x²
```

```
True
```

```
True
```

To /.{lysh, y'[x]Øy1[x]} **shmaínei** ó**ti sugcr**ó**nwV antikaqisto**ú**me thn** y[x] **me** Ą−1êx sta8era@1D **kai thn** y'[x] **me** y1[x] **m**é**sa sthn** eqn. **AV k**á**noume kai** é**na apl**ó **par**á**deigma gia na katal**á**boume thn diajor**á:

```
8a, b, c< ê. a → b ê. b → d

8d, d, c<
```

```
8a, b, c< ê. 8a → b, b → d<

8b, d, c<
```

Sto 1**o par**á**deigma antikatast**ή**same diadocik**á **en**ώ **sto** 2**o k**á**name mia antikat**á**stash sugcr**ó**nwV**! **Giaut**ó **ex**á**llou pro**é**kuyan kai diajoretik**á **apotel**é**smata**!
Gia na katalá**bei to** *Mathematica* ó**ti me** y[x] **ennoo**ú**me mia sun**á**rthsh tou** x **qa mporo**ú**same gia par**á**deigma na or**í**soume sth q**é**sh thV** y **thn** f[z_]:=y[x]/.soll[[1]]/.xØz **kai sth sun**é**ceia** ó**pou up**á**rcei h** y **na th antikaqisto**ú**me me thn** f :

```
f@z_D := y@xD ê. soll@@1DD ê. x → z
f'@zD
eqn ê. y → f
```

```
 A^(-1êz) sta8era@1D
 ¡¡¡¡¡¡¡¡¡¡¡¡¡¡¡¡¡¡¡¡¡¡¡¡¡¡¡¡¡¡¡¡
        z^2
```

```
True
```

Scólio: An antí y[x] **qésoume aplá** y **mésa sthn** DSolve **tóte ta prágmata aplousteúontai. Proséxte prosektiká ta apotelésmata parakátw**:

```
y@xD ê. soll@@1, 1DD
D@y@xD ê. soll@@1, 1DD, xD
y@zD ê. soll@@1, 1DD
D@y@zD ê. soll@@1, 1DD, zD
Clear@yD
soll0 = DSolve@eqn, y, xD
y@3D ê. soll0@@1, 1DD
y@zD ê. soll0@@1, 1DD
D@y@zD ê. soll0@@1, 1DD, zD


Ą^{-1êx} sta8era@1D
```

```
Ą^{-1êx} sta8era@1D
-------------------
        x^2
```

```
y@zD
```

```
yÅ@zD
```

```
88y → Function@8x<, Ą^{-1êx} C@1DD<<
```

```
C@1D
----
Ą^{1ê3}
```

```
Ą^{-1êz} C@1D
```

```
Ą^{-1êz} C@1D
-------------
     z^2
```

prosocή ά**llo apot**έ**lesma me thn** DSolve[eqn,y,x] **kai** ά**llo** DSolve[eqn,y[x],x]. **H** DSolve[eqn,y,x] **e**ί**nai pol**ύ **cr**ή**simh di**ό**ti maV d**ί**nei to** y **na e**ί**nai sun**ά**rthsh tou** x **kai o t**ύ**poV thV** y **e**ί**nai** y[x]/.soll0[[1,1]] **dhl**. =Ą^{-1êx} C@1D. ¨**Etsi gia par**ά**deigma to** y[z] **e**ί**nai** ί**so me** y[z]/.soll0[[1,1]] **dhl**. Ą^{-1êz} C@1D**kai h par**ά**gwgoV wV proV** z **e**ί**nai** D[y[z]/.soll0[[1,1]], z] **dhl**. Ą^{-1êz} C@1D / z^2 . **Parathre**ί**ste** ό**ti to** *Mathematica* **gia thn ant**ί**stoich ap**ά**nthsh pou d**ί**nei h** DSolve[eqn,y[x],x] , **mpore**ί **na bre**ί **thn par**ά**gwgo wV proV** x **all**ά **den gnwr**ί**zei pwV na brei to** y[z] **o**ύ**te thn par**ά**gwgo wV proV** z(ό**pwV** ή**dh proanaj**έ**rame**)!!!

To pleonέ**kthma e**ί**nai janer**ό: **mporo**ύ**me me thn l**ύ**sh gia thn** ά**gnwsth** y **pou d**ί**nei h** DSolve[eqn,y,x] **na k**ά**noume e**ύ**kola thn dokim**ή:

```
lysh1 = soll0@@1, 1DD
eqn ê. lysh1

y → Function@8x<, Ą^-1êx C@1DD
```

```
True
```

8.2 Oi dunatóthteV thV DSolve

Geniká gia thn DSolve prépei na xéroume óti epilúei 1) óleV tiV gram. exisώseiV me staqeroúV suntelestéV poiasdήpote táxhV 2)éna eurú jásma grammikώn exisώsewn me mh staqeroúV suntelestéV mécri 2hV táxhV kai 3) éna eurú jásma mh grammikώn diajorikώn exisώsewn. Mporeí epíshV na epilúsei diajorikéV exisώseiV me merikéV paragώgouV arkei na dώsoume tiV anexárthteV metablhtéV apo tiV opoíeV exartátai h zhtoúmenh sunárthsh, upó morjή lístaV. Akolouqoún paradeígmata:

Grammikή diajorikή exíswsh 1hV táxhV

```
eqn1 = y '@xD + y@xD ê x + (Cos@xD − E^x)/x « 0
soll1 = DSolve@eqn1, y, xD

(−Ą^x + Cos@xD)/x + y@xD/x + yÅ@xD == 0
```

```
99y → FunctionA8x<, C@1D/x + (Ą^x − Sin@xD)/x E==
```

Eúresh miaV timήV thV y p.c thV y[3]

```
y@3D ê. soll1@@1, 1DD

C@1D/3 + 1/3 HĄ^3 − Sin@3DL
```

Όtan upárcoun arcikéV sunqήkeV tiV anagrájoume mésa sthn entolή DSolve me morjή lístaV sth morjή:

DSolve[{eqn, sunqήkeV},y,x]

P. c gia na luqeí h eqn1 me thn arcikή sunqήkh y[2]==1 tóte qa grájoume:

```
DSolve@8eqn1, y@2D « 1<, y, xD
```

```
99y → FunctionA8x<, (2 − Ą^2 + Ą^x + Sin@2D − Sin@xD)/x E==
```

Grammikή diajorikή exίswsh 2hV tάxhV me staqeroύV suntelestέV

```
eqn2 = y''@xD − 2 y'@xD + y@xD « 0
soll1 = DSolve@eqn2, y, xD
```

```
y@xD − 2 y^Å@xD + y^ÅÅ@xD == 0
```

```
88y → Function@8x<, Ą^x C@1D + Ą^x x C@2DD<<
```

Edώ emjanίzontai dύo staqerέV sthn lύsh. Gia na exajanίsoume thn mia apo tiV dύo qa prέpei na dώsoume kάpoieV arcikέV sunqήkeV sthn y ή sthn parάgwgo y'. An dώsoume arcikέV sunqήkeV sugcrόnwV kai stiV dύo tόte paίrnoume mia lύsh y cwrίV staqerέV:

```
DSolve@8eqn2, y@0D « 1<, y, xD
DSolve@8eqn2, y'@1D « 0<, y, xD
DSolve@8eqn2, y@0D « 1, y'@1D « 0<, y, xD
```

```
88y → Function@8x<, Ą^x H1 + x C@2DLD<<
```

```
88y → Function@8x<, Ą^x H−2 + xL C@2DD<<
```

```
99y → FunctionA8x<, −(1/2) Ą^x H−2 + xLE==
```

Grammikή diajorikή exίswsh 2hV tάxhV me mh staqeroύV suntelestέV

```
eqn3 = x y''@xD − H2 x + 1L y'@xD + Hx + 1L y@xD « x è!!!x E^x
DSolve@eqn3, y, xD
```

```
H1 + xL y@xD − H1 + 2 xL y^Å@xD + x y^ÅÅ@xD == Ą^x x^3ê2
```

```
99y → FunctionA8x<, (4/5) Ą^x x^5ê2 + Ą^x C@1D + (1/2) Ą^x x^2 C@2DE==
```

Exíswsh me merikéV paragógouV wV proV x **kai** y **kai ágnwsth sunárthsh thn** z[x,y]

```
eqn4 = x D@z@x, yD, xD + y D@z@x, yD, yD « z@x, yD
soll4 = DSolve@eqn4, z, 8x, y<D

y z^H0,1L@x, yD + x z^H1,0L@x, yD == z@x, yD
```

```
99z → FunctionA8x, y<, x C@1DA y/x EE==
```

Prosocή edώ me C@1D@ y/x D**ennooúme mia sunárthsh** C[1] **miaV metablhtήV éstw** t **sthn opoía écoume antikatastήsei thn metablhtή** t **me to lógo** y/x. **AV kánoume kai dokimή**

```
soll4@@1, 1DD
eqn4 ê. soll4@@1, 1DD
eqn4 ê. soll4@@1, 1DD êê Simplify

z → FunctionA8x, y<, x C@1DA y/x EE
```

```
y C@1D^ÅA y/x E + x ( C@1DA y/x E − y C@1D^Å@ y/x D / x ) == x C@1DA y/x E
```

```
True
```

H Simplify **kánei touV apaitoúmenouV upologismoúV kai aplopoieí thn parástash**

OmogenήV grammikή exíswsh 2**hV táxhV** (**me staqeroúV suntelestéV kai**)**me merikéV paragógouV kai ágnwsth sunárthsh thn** z[x,y]

```
eqn5 =
 D@z@x, yD, 8x, 2<D − 5 D@z@x, yD, x, yD + 6 D@z@x, yD, 8y, 2<D « 0
soll5 = DSolve@eqn5, z, 8x, y<D

6 z^H0,2L@x, yD − 5 z^H1,1L@x, yD + z^H2,0L@x, yD == 0
```

```
88z → Function@8x, y<, C@1D@3 x + yD + C@2D@2 x + yDD<<
```

Edώ blépoume óti h genikή lúsh periécei duo sunartήseiV C[1] **kai** C[2] **stiV opoíeV écoume antikatastήsei thn metablhtή touV me** 3 x+y **kai me** 2 x+y **antístoica**. H **dokimή gínetai eúkola**:

```
soll5@@1, 1DD
eqn5 ê. soll5@@1, 1DD
eqn5 ê. soll5@@1, 1DD êê Simplify

z → Function@8x, y<, C@1D@3 x + yD + C@2D@2 x + yDD
```

```
9 C@1DÄÄ@3 x + yD + 4 C@2DÄÄ@2 x + yD + 6 HC@1DÄÄ@3 x + yD + C@2DÄÄ@2 x + yDL −
   5 H3 C@1DÄÄ@3 x + yD + 2 C@2DÄÄ@2 x + yDL == 0
```

```
True
```

8.3 Sustήmata diajorikώn exisώsewn kai h entolή FullSimplify

Me thnDSolve **mporoύme na epilύsoume kai sustήmata diajorikώn exisώsewn**, **aplά qa prosέxoume na dώsoume mazί me tiV diajorikέV exisώseiV kai tiV arcikέV** ή **oriakέV sunqήkeV mέsa se mia lίsta kai tiV άgnwsteV sunartήseiV mέsa se άllh lίsta**.**P**.**c**

```
system6 =
  8y '@xD + y@xD − z@xD « Sin@xD, z '@xD + y@xD + z@xD « Cos@xD<;
agnwstesSynarthseis6 = 8y@xD, z@xD<;
soll6 = DSolve@system6, agnwstesSynarthseis6, xD
soll6 êê Simplify
soll6 êê FullSimplify

88y@xD → Ą−x C@1D Cos@xD + Ą−x C@2D Sin@xD +
      Ą−x Sin@xD HĄx Cos@xD2 + Ąx Sin@xD2L, z@xD → Ą−x C@2D Cos@xD −
      Ą−x C@1D Sin@xD + Ą−x Cos@xD HĄx Cos@xD2 + Ąx Sin@xD2L<<
```

```
88y@xD → Ą−x HC@1D Cos@xD + HĄx + C@2DL Sin@xDL,
   z@xD → Ą−x HHĄx + C@2DL Cos@xD − C@1D Sin@xDL<<
```

```
88y@xD → Sin@xD + Ą−x HC@1D Cos@xD + C@2D Sin@xDL,
   z@xD → Cos@xD + Ą−x HC@2D Cos@xD − C@1D Sin@xDL<<
```

Edώ crhsimopoiήsame kai thn sunάrthsh FullSimplify **gia na kάnoume όso to dunatόn perissόtereV aplopoiήseiV**. **AlliώV h lush όpwV blέpoume έcei perίplokh morjή**. **To ίdio qa kάnoume όpou creiάzetai kai parakάtw**. **Gia thn dokimή qa prέpei na broύme kai tiV paragώgouV** y'[x] **kai** z'[x] **kai na antikatastήsoume**:

```
system6 ê. soll6@@1DD ê. D@soll6@@1DD, xD
system6 ê. soll6@@1DD ê. D@soll6@@1DD, xD êê Simplify

8Ą^-x Sin@xD HĄ^x Cos@xD^2 + Ą^x Sin@xD^2L == Sin@xD,
 Ą^-x Cos@xD HĄ^x Cos@xD^2 + Ą^x Sin@xD^2L == Cos@xD<
```

```
8True, True<
```

Scólio An eícame qései parapánw agnwstesSynarthseis6={y, z} **antí** agnwstesSynarthseis6={y[x],z[x]} **qa eícame apojúgei na broúme kai tiV paragógouV twn** y **kai** z **gia na kánoume thn dokimή** . **Autó ómwV den to káname dióti h apánthsh pou qa maV dinótan qa perieíce wV gnwstó**, **ta** yØFunction[{x},... **kai** zØFunction[{x},...**kai dustucóV tóte to** *Mathematica* **adunateí na kánei** Simplify ή FullSimplify!!!!

An sto parapánw sústhma écoume kai arcikéV sunqήkeV p.c y[**p**/2]=1 **kai** z[**p**/2]=0 **tóte autéV prostíqontai polú aplá sto sústhma**:

```
DSolve@8system6, y@Pi ê 2D « 1, z@Pi ê 2D « 0<,
  agnwstesSynarthseis6, xD êê FullSimplify

88y@xD → Sin@xD, z@xD → Cos@xD<<
```

8.4 H entolή NDSolve

Kleínoume to kejálaio me thn NDSolve **h opoía parécei th dunatóthta ariqmhtikήV epílushV diajorikón exisósewn kai susthmátwn**. **SunήqwV crhsimopoioúme thn** NDSolve ó**pou mia akribή lúsh den mporeí na breqeí me thn** DSolve. **P.c**

```
Clear@eqn, sollD
eqn = y''@xD + 5 Log@y@xDD « 0;
soll « DSolve@8eqn, y'@0D « 1, y@0D « 1<, y@xD, xD

DSolve::bvimp :
 General solution contains implicit solutions. In the boundary value problem
   these solutions will be ignored, so some of the solutions will be lost.

soll == 8<
```

DSolve::bvimp: General solution contains implicit solutions. In the boundary value problem these solutions will be ignored, so some of the solutions will be lost.

soll=={}

H **súntaxh thV entolήV eínai**

NDSolve[**diajorik**έV **exis**ώ**seiV**,y[x],{x,xmin,xmax}]

An upά**rcoun periss**ό**tereV apo mia diaj**. **exis**ώ**seiV mpa**ί**noun se l**ί**sta maz**ί **me tiV arcik**έ**V sunq**ή**keV an aut**έ**V b**έ**baia up**ά**rcoun**. **Anti** y[x] **mporo**ύ**me na gr**ά**youme pio apl**ά y. **Fusik**ά ό**pwV** ή**dh x**έ**roume sthn de**ύ**terh per**ί**ptwsh qa p**ά**roume thn** y **wV sun**ά**rthsh tou** x **en**ώ **sthn pr**ώ**th per**ί**ptwsh qa p**ά**roume thn tim**ή y[x] **thV** y **se morj**ή **kan**ό**na**. **Sto t**έ**loV pr**έ**pei na dhl**ώ**soume kai to di**ά**sthma** {x,xmin,xmax}, **m**έ**sa sto opo**ί**o q**έ**loume na doqe**ί **h** (**proseggistik**ή) **l**ύ**sh**. **P.c**

```
Clear@sollD
soll = NDSolve@8eqn, y'@0D « 1, y@0D « 1<, y, 8x, 0, 4<D
soll1 = NDSolve@8eqn, y'@0D « 1, y@0D « 1<, y@xD, 8x, 0, 4<D

88y → InterpolatingFunction@880., 4.<<, <>D<<
```

```
88y@xD → InterpolatingFunction@880., 4.<<, <>D@xD<<
```

H NDSolve **energe**ί **wV ex**ή**V**: **Upolog**ί**zei th l**ύ**sh** y, **proseggistik**ά, **pr**ώ**ta gia** έ**na mikr**ό **pl**ή**qoV shme**ί**wn tou diast**ή**matoV** {xmin,xmax}. **Aut**ά **ta shme**ί**a maz**ί **me tiV prok**ύ**ptouseV tim**έ**V ta b**ά**zei se mia l**ί**sta pou thn onom**ά**zei gia suntom**ί**a** <>. **Sth sun**έ**ceia k**ά**nei kat**ά**llhlh parembol**ή **stiV parap**ά**nw tim**έ**V thV l**ί**staV** <> **gia na mpor**έ**sei na bre**ί **thn tim**ή **thV** y[x] **gia opoiod**ή**pote** ά**llo shme**ί**o** x **tou diast**ή**matoV**. **Gia aut**ό ά**llwste sthn ap**ά**nthsh h sun**ά**rthsh** y **anaj**έ**retai wV** Interpolating-Function **dhlad**ή **mia sun**ά**rthsh pou oi tim**έ**V thV** y[x] **upolog**ί**zontai me thn m**έ**qodo parembol**ή**V**. **An q**έ**loume na bro**ύ**me gia par**ά**deigma thn tim**ή y[0.02] **qa gr**ά**youme** έ**na apo ta parak**ά**tw**

```
Hy ê. soll@@1, 1DDL@0.02D
soll@@1, 1, 2DD@0.02D
y@xD ê. soll1 ê. x → 0.02

1.01999
```

```
1.01999
```

```
81.01999<
```

Prosocή **den gr**ά**yame sk**έ**ta** y/.soll[[1,1]][0.02] ή y/.soll[[1]]/.x->0.02 **di**ό**ti den qa doul**έ**yei**! **Genik**ά **me** `InterpolatingFunction[ … ][x]` **br**ί**skoume thn tim**ή **thV sun**ά**rthshV parembol**ή**V** y **sto** x. **Ep**ί**shV mporo**ύ**me na bro**ύ**me kai** ό**seV** ά**lleV tim**έ**V q**ά**loume kai na k**ά**noume kai grajik**ή **par**ά**stash thV** (**proseggistik**ή**V**) **l**ύ**shV thV diajorik**ή**V ex**ί**swshV**. **P.c an qeloume na bro**ύ**me tiV tim**έ**s** y[x] **gia** x=0, 0.08, 2 0.08, 3 0.08, ... **qa gr**ά**youme**

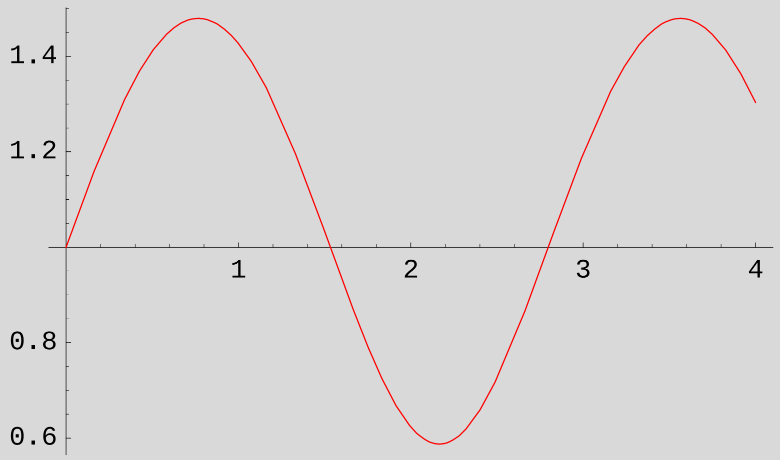
```
pinakas1 = Table@y@xD ê. soll1, 8x, 0, 4, .08<D
pinakas = Table@soll@@1, 1, 2DD@xD, 8x, 0, 4, .08<D
```

```
881.<, 81.07958<, 81.15673<, 81.22925<, 81.29518<, 81.35286<,
 81.40089<, 81.43816<, 81.46382<, 81.47731<, 81.47834<,
 81.46688<, 81.44318<, 81.40776<, 81.36143<, 81.30524<,
 81.24056<, 81.169<, 81.09246<, 81.01311<, 80.933368<,
 80.855841<, 80.783302<, 80.718571<, 80.664395<, 80.623265<,
 80.597208<, 80.587575<, 80.594882<, 80.618737<, 80.657895<,
 80.710411<, 80.773845<, 80.845475<, 80.92248<, 81.00208<,
 81.08163<, 81.15869<, 81.23106<, 81.2968<, 81.35424<,
 81.40201<, 81.43898<, 81.46433<, 81.4775<, 81.4782<,
 81.46642<, 81.44241<, 81.4067<, 81.3601<, 81.30367<<
```

```
81., 1.07958, 1.15673, 1.22925, 1.29518, 1.35286, 1.40089,
 1.43816, 1.46382, 1.47731, 1.47834, 1.46688, 1.44318,
 1.40776, 1.36143, 1.30524, 1.24056, 1.169, 1.09246, 1.01311,
 0.933368, 0.855841, 0.783302, 0.718571, 0.664395, 0.623265,
 0.597208, 0.587575, 0.594882, 0.618737, 0.657895, 0.710411,
 0.773845, 0.845475, 0.92248, 1.00208, 1.08163, 1.15869,
 1.23106, 1.2968, 1.35424, 1.40201, 1.43898, 1.46433,
 1.4775, 1.4782, 1.46642, 1.44241, 1.4067, 1.3601, 1.30367<
```

Oi timέV den diajέroun se kάqe perίptwsh aplώV ston prώto pίnaka emjanίzontai me thn morjή lίstaV. Me **thn** Plot **mporoύme na kάnoume thn grajikh thV parάstash**:

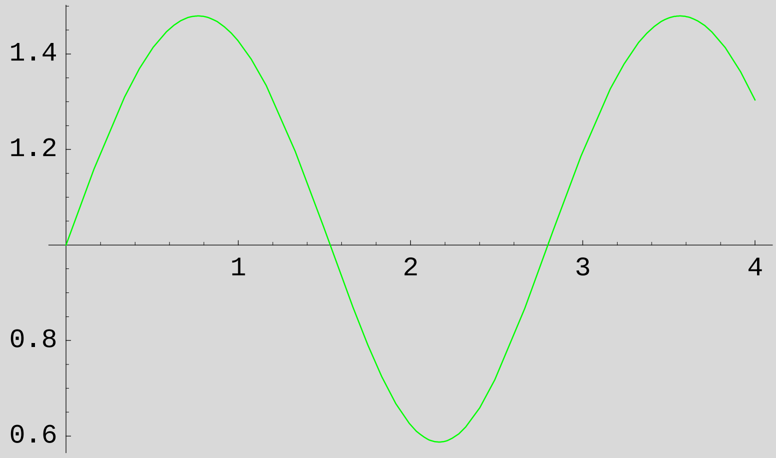
```
Plot@y@xD ê. soll1, 8x, 0, 4<, PlotStyle → RGBColor@1, 0, 0DD
```




```
¦ Graphics ¦
```

Fusikά gia thn deύterh perίptwsh qa mporoύsame na grάyoume

```
Plot@soll@@1, 1, 2DD@xD, 8x, 0, 4<, PlotStyle → RGBColor@0, 1, 0DD
```




```
¦ Graphics ¦
```

An tώra kάnoume klίk pάnw sto scήma kai sthn sunέceia patήsoume sunecώV to plήktro Alt mporoύme na doύme sthn aristerή gwnίa thV oqόnhV maV tiV suntetagmέneV twn shmeίwn thV kampύlhV maV. H entolή NDSolve ejarmόzetai exίsou apotelesmatikά kai se sustήmata diajorikώn exisώsewn.

Άskhsh: Na luqeί ariqmhtikά to sύsthma twn diajorikώn exisώsewn y'[x]==Log[z[x]]+y[x],z'[x]==-Log[y[x]]-2 z[x]me arcikέV sunqήkeV y[2]==z[2]==1 sto diάsthma [2,5]. Na breqoύn oi timέV twn sunartήsewn y[x] kai z[x] sta shmeίa x=2,2.5,3,3.5,...,5 kai na gίnoun oi grajikέV touV parastάseiV.