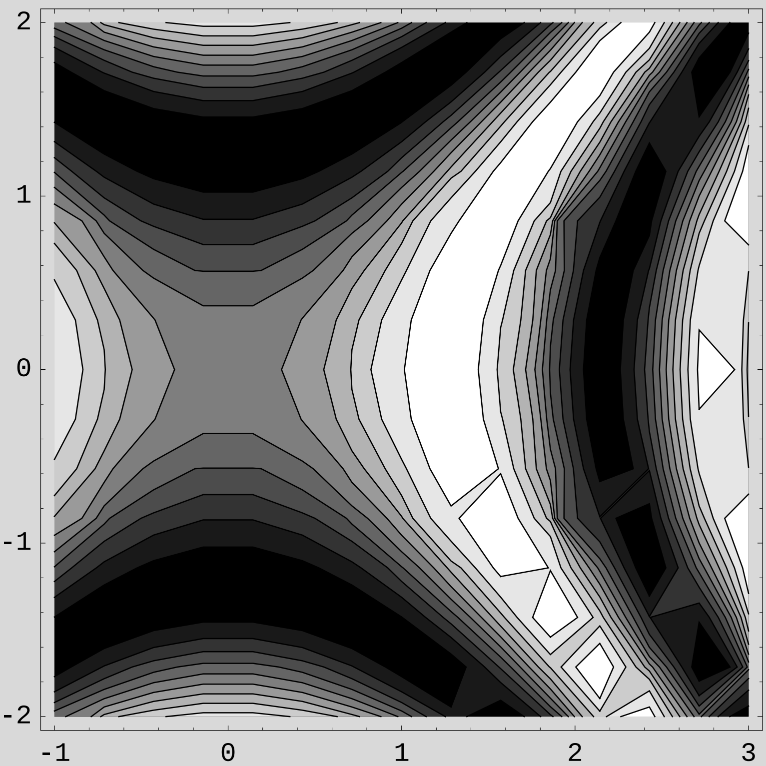
9.2 Μελετώντας τρισδιάστατα γραφικά στο επίπεδο

9.2.1 Oi sunartήseiV Contour Plot kai DensityPlot

Me thn ContourPlot[f[x,y], {x,xmin,xmax},{y,ymin,ymax}] **scediάzoume thn** f[x,y] **pάnw sto epίpedo** Oxy, **dίnontaV sto shmeίo** (x,y) έ**na cr**ώ**ma** (**sun**ή**qwV ap**ό**crwsh tou gkr**ί**zou**) **pou antistoice**ί **sthn tim**ή f[x,y]. T**a shme**ί**a pou** έ**coun megal**ύ**terh tim**ή f[x,y] **e**ί**nai pio jwteinά en**ώ **aut**ά **pou** έ**coun mikr**ό**terh e**ί**nai pio skoteinά**. **P**.**c**

```
ContourPlot@Sin@x^2 − y^2D, 8x, −1, 3<, 8y, −2, 2<D;
```



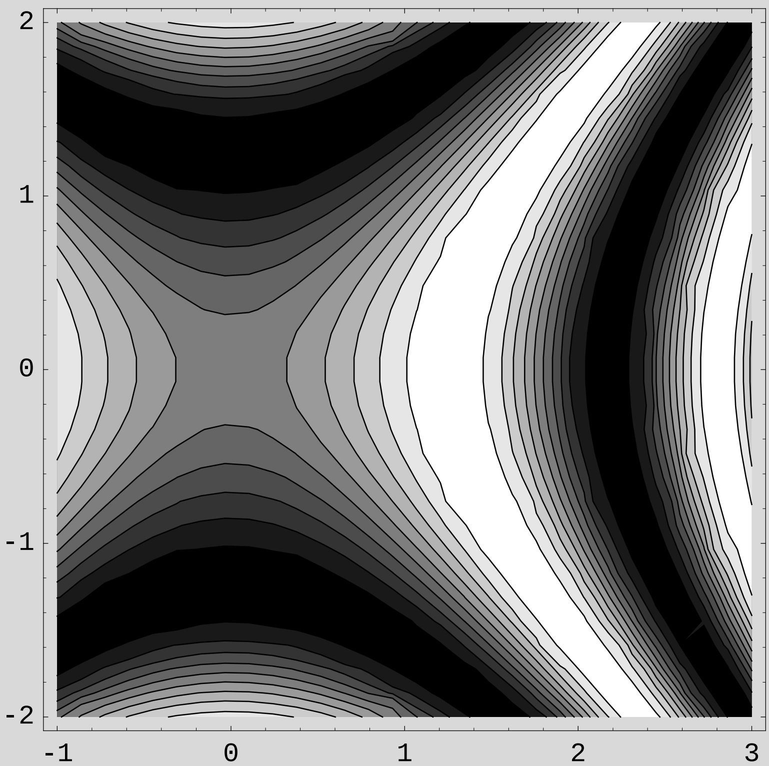
Parathroύ**me** 10 **apocr**ώ**seiV tou gr**ί(**to leuk**ό **den to metr**ά**me**, **ousiastik**ά έ**coume maz**ί **me to leuk**ό 11). **H na to po**ύ**me diajoretik**ά: **To ped**ί**o tim**ώ**n**(**ston** ά**xona O**z) έ**cei cwriste**ί **se** 10+1 ί**sou m**ή**kouV diast**ή**mata** έ**stw D**1,**D**2,..,**D**11. **K**ά**qe di**ά**sthma pa**ί**rnei** έ**na cr**ώ**ma tou gkr**ί **xekin**ώ**ntaV apo to ma**ύ**ro**. Ό**sa shme**ί**a** (x,y) **tou epip**έ**dou apeikon**ί**zontai**(**m**έ**sw thV** f) **m**έ**sa sto** ί**dio di**ά**sthma D**$_i$ **qa p**ά**roun thn** ί**dia ap**ό**crwsh**! '**Etsi p**ά**nw sto ep**ί**pedo O**xy **emjan**ί**zontai crwmatik**έ**V lwr**ί**deV**,**ta isouy**ή **ep**ί**peda**, **ta opo**ί**a diacwr**ί**zontai metax**ύ **touV apo k**ά**poieV kamp**ύ**leV pou l**έ**gontai isouye**ί**V**(Contours). Ό**la ta shme**ί**a miaV isouy**ή **kamp**ύ**lhV pa**ί**rnoun thn** ί**dia akrib**ώ**V tim**ή **me thn** f ! **Fusik**ά **epeid**ή **up**ά**rcoun** ά**peira shme**ί**a** (x,y) **m**έ**sa sto orqog**ώ**nio scediasmo**ύ D={x,xmin,xmax}X{y,ymin,ymax}, **to** *Mathematica* **qa dial**έ**xei deigmatoleiptik**ά **l**ί**ga shme**ί**a apo to** D **kai me b**ά**sh tiV tim**έ**V touV qa scedi**ά**sei tiV isouye**ί**V kamp**ύ**leV**. **Ta shme**ί**a aut**ά **l**έ**gontai** PlotPoints. **Fusik**ά **to apot**έ**lesma pou pa**ί**rnoume** έ**cei** ό**pwV bl**έ**poume poll**έ**V at**έ**leieV**. **Gia par**ά**deigma oi isouye**ί**V kamp**ύ**leV den e**ί**nai** ό**so qa perim**έ**name omal**έ**V**. **Ep**ί**shV mporo**ύ**me na parathr**ή**soume** ό**ti h leuk**ή **lwr**ί**da sta dexi**ά **e**ί**nai sto k**ά**tw m**έ**roV thV kommatiasm**έ**nh**! **Aut**ό **wje**ί**letai kur**ί**wV sto gegon**ό**V** ό**ti h proepilegm**έ**nh tim**ή **tou** PlotPoints **e**ί**nai** 15. **Op**ό**te apo to di**ά**sthma** D **epil**έ**gontai** 15X15 **to pl**ή**qoV shme**ί**a pou den e**ί**nai arket**ά **an h** f[x,y] έ**cei ap**ό**tomeV "lako**ύ**beV" kai "loj**ί**skouV" sto** D. **Qa prospaq**ή**soume na antimetwp**ί**soume aut**έ**V tiV at**έ**leieV**. AV **do**ύ**me** ό**mwV pr**ώ**ta**, **poi**έ**V e**ί**nai oi epilog**έ**V thV** ContourPlot:

```
Options@ContourPlotD

8AspectRatio → 1, Axes → False, AxesLabel → None,
 AxesOrigin → Automatic, AxesStyle → Automatic,
 Background → Automatic, ColorFunction → Automatic,
 ColorFunctionScaling → True, ColorOutput → Automatic,
 Compiled → True, ContourLines → True, Contours → 10,
 ContourShading → True, ContourSmoothing → True,
 ContourStyle → Automatic, DefaultColor → Automatic, Epilog → 8<,
 Frame → True, FrameLabel → None, FrameStyle → Automatic,
 FrameTicks → Automatic, ImageSize → Automatic, PlotLabel → None,
 PlotPoints → 15, PlotRange → Automatic, PlotRegion → Automatic,
 Prolog → 8<, RotateLabel → True, Ticks → Automatic,
 DefaultFont Ç $DefaultFont, DisplayFunction Ç $DisplayFunction,
 FormatType Ç $FormatType, TextStyle Ç $TextStyle<
```

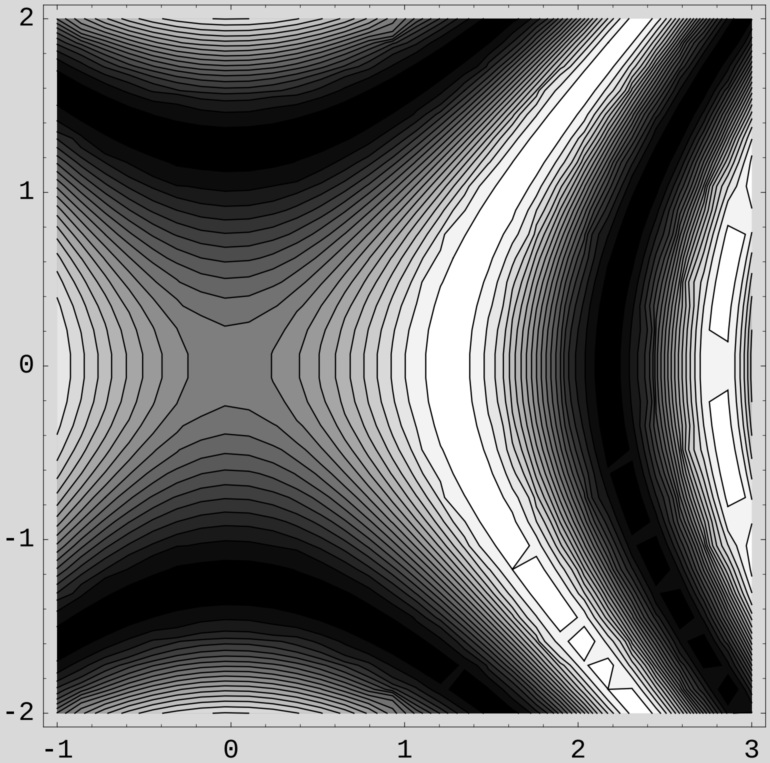
AllάzontaV kάpoia apo ta parapάnw carakthristikά mporoύme na έcoume έna kalό apotέlesma. P.c mporoύme na epitrέyoume sto *Mathematica* **na kάnei kalύterh deigmatoleiyίa paίrnontaV perissόtera shmeίa. WV apotέlesma qa έcoume pio akribeίV isouyeίV kampύleV. P.c**

```
gr = ContourPlot@Sin@x^2 - y^2D,
   8x, -1, 3<, 8y, -2, 2<, PlotPoints → 30D;
```



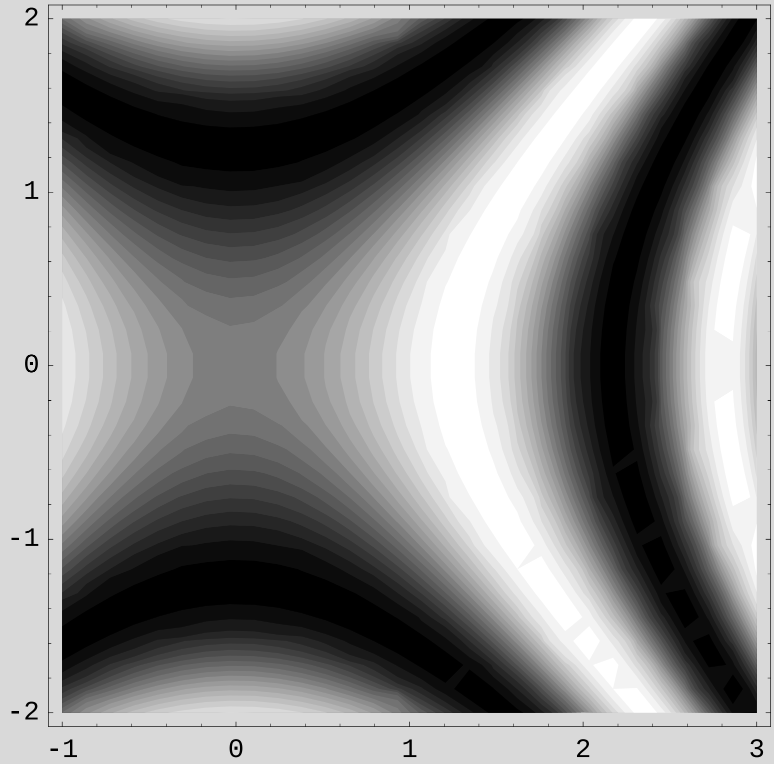
Mporoύme epίshV na zhtήsoume perissόtereV isouyeίV(kai άra perissόtereV apocrώseiV) megalώnontaV thn proepilegmέnh timή tou Contours **pou eίnai** 10(ή **akribέstera** 10+1 **όpwV proanajέrame). P.c**

```
gr1 = Show@gr, Contours → 20D;
```



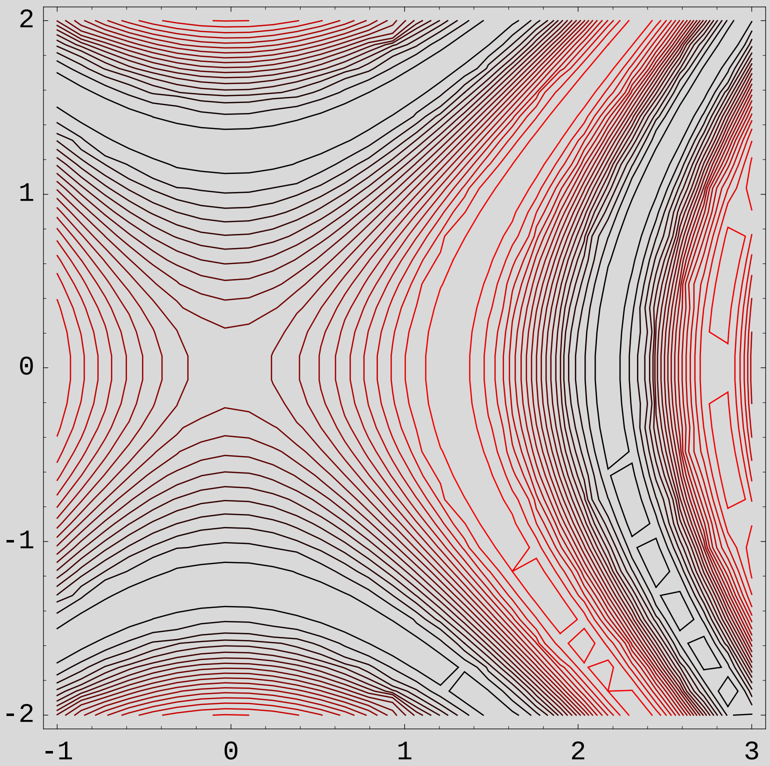
Prosέxte ό**ti auxά nontaV to pl**ή**qoV twn** Contours **c**ά**netai h akr**ί**beia sto scediasm**ό **twn isouy**ώ**n**!!! Ά**ra qa pr**έ**pei na aux**ή**soume xan**ά **ta** PlotPoints **gia na pet**ύ**coume thn akr**ί**beia ston scediasm**ό! **An t**ώ**ra q**έ**loume m**ό**no tiV diabaqm**ί**seiV cwr**ί**V tiV isouye**ί**V qa gr**ά**jame** ContourLines->False **p.c**

```
gr2 = Show@gr1, ContourLines → FalseD;
```



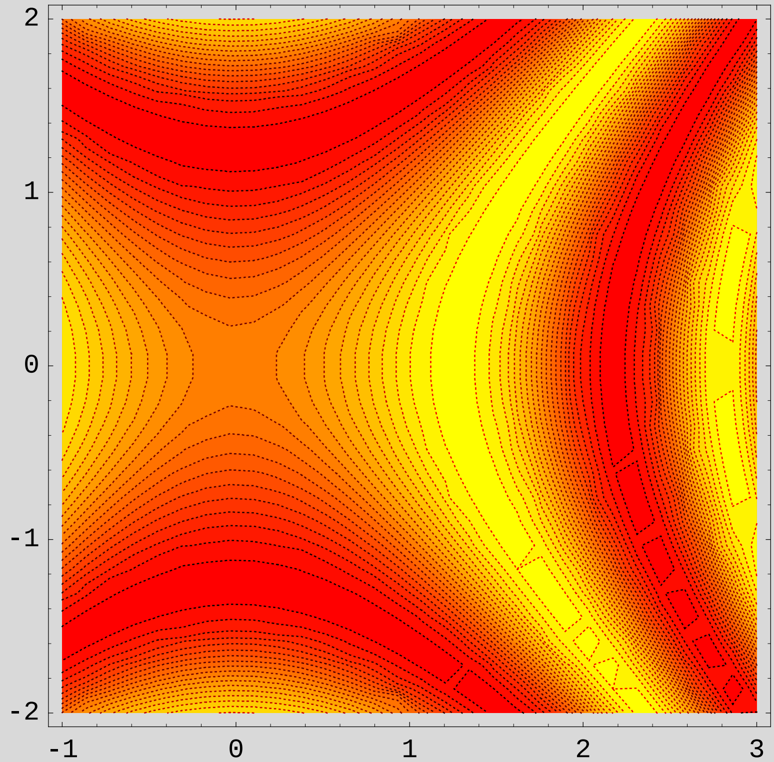
Sígoura polú pio katanohtó apotélesma! MerikéV jorέV den maV endiajέroun tóso oi apocrώseiV óso oi ídieV oi isouyeίV! Parakάtw dίnoume έna tέtoio parάdeigma. Me ContourShading->False exajanίzoume tiV apocώseiV enώ me ContourStyle->({RGBColor[#,0,0]}&)/@Range[0,0.95,.05] dίnoume 20 diajoretikέV apocrώseiV(ósa eίnai kai ta Contours) tou kókkinou stiV antίstoiceV isouyeίV kampúleV.

```
gr3 = Show@gr1, ContourShading -> False,
  ContourStyle → H8RGBColor@#, 0, 0D< &L ê@ Range@0, 0.95, .05DD;
```



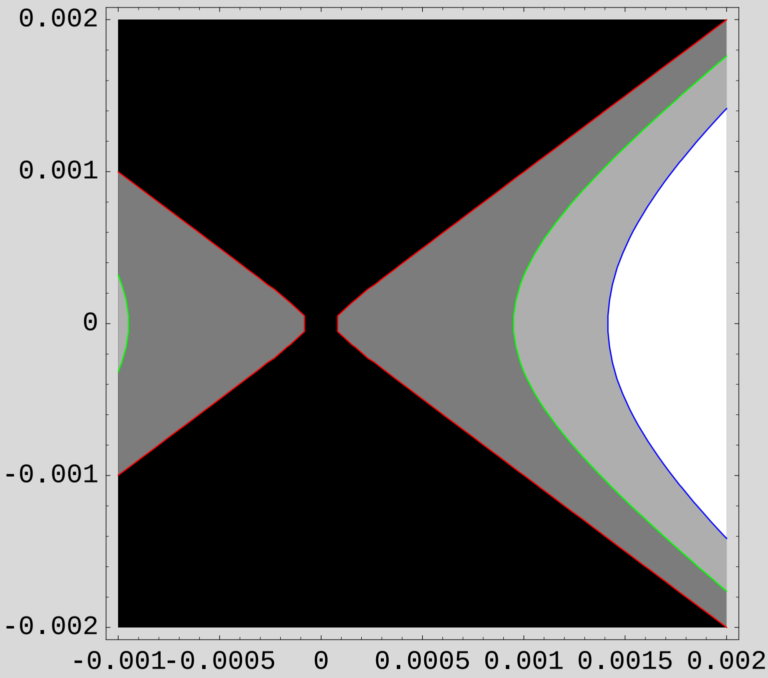
Edώ me έntono kόkkino eίnai oi isouyeίV pou brίskontai pio yhlά apo tiV άlleV. Qa mporoύsame tώra na emjanίsoume kai ta isouyή epίpeda me diabaqmίseiV tou kίtrinou-kόkkinou(me thn boήqeia thV Color-FunctionØ(RGBColor[1,#,0]&)) kai tiV ContourLines kόkkineV kai diakekommέneV p.c

```
gr4 = Show@gr1, ContourShading → True,
  ColorFunction → HRGBColor@1, #, 0D &L,
  ContourStyle → H8RGBColor@#, 0, 0D, Dashing@80.0015, 0.005<D< &L ê@
   Range@0, 0.95, .05DD;
```



Den prέpei na xecάsoume na anajέroume thn Contours->{z1,z2,z3,...} me thn opoίa epilέgoume na mpoύn isouyeίV mόno stiV sugkekrimmέneV timέV tou z. P.c qa prospaqήsoume na diereunίsoume thn f kontά sto shmeίo (0,0) meletώntaV mόno kάpoieV qetikέV isouyeίV me timέV kontά sto f[x,y]=0 p.c :ContoursØ84 10^{-9}, 9 10^{-7}, 2 10^{-6}<Gia akrίbeia megalώnoume kai to plήqoV twn deigmatoleiptikώn shmeίwn(PlotPointsØ40)

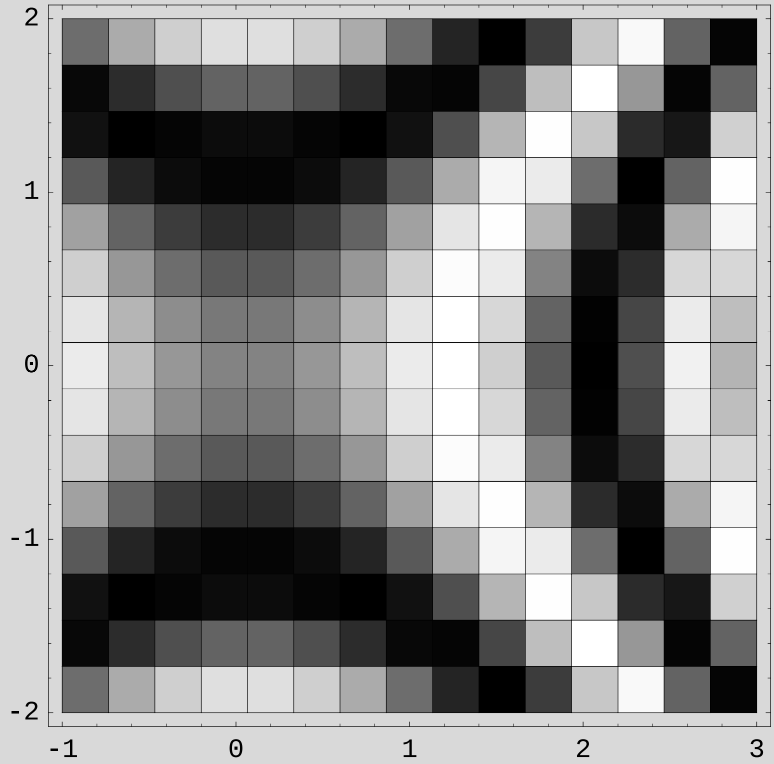
```
gr5 =
  ContourPlot@Sin@x^2 - y^2D, 8x, -0.001, 0.002<, 8y, -0.002, 0.002<,
   Contours → 84 10^-9, 9 10^-7, 2 10^-6<, PlotPoints → 40, ContourStyle →
    88RGBColor@1, 0, 0D<, 8RGBColor@0, 1, 0D<, 8RGBColor@0, 0, 1D<<D;
```



Me maύro crώma eίnai όleV oi timέV thV sunάrthshV <$4\ 10^{-9}$, **me endiάmeso gkrί oi timέV metaxύ** $4\ 10^{-9}$ και $9\ 10^{-7}$**k.o.k**

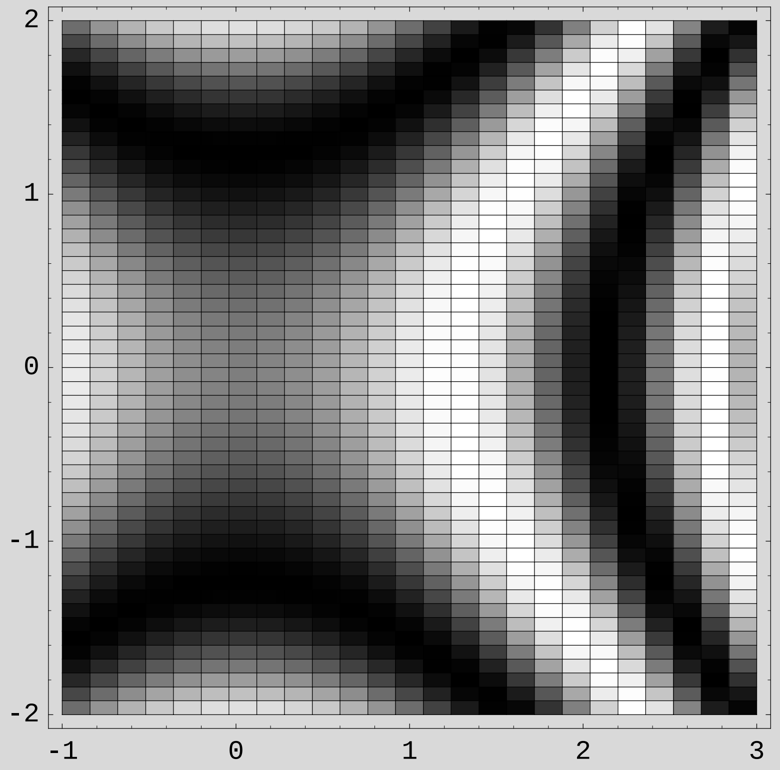
H DensityPlot den prospaqeί na scediάsei kάpoieV isouyeίV kampύleV όpwV h ContourPlot. **Apl ώV parάgei έna plέgma**(mesh)**kai kάpoieV apocrώseiV mέsa se autό. H proepilegmέneV apocrώseiV eίnai tou gkrί. Skoteinά gkrί crhsimopoioύntai gia baqoulώmata thV** f[x,y] **dhl. gia mikrέV timέV kai anoikta gkrί gia megάleV timέV. P.c**

```
DensityPlot@Sin@x^2 - y^2D, 8x, -1, 3<, 8y, -2, 2<D;
```



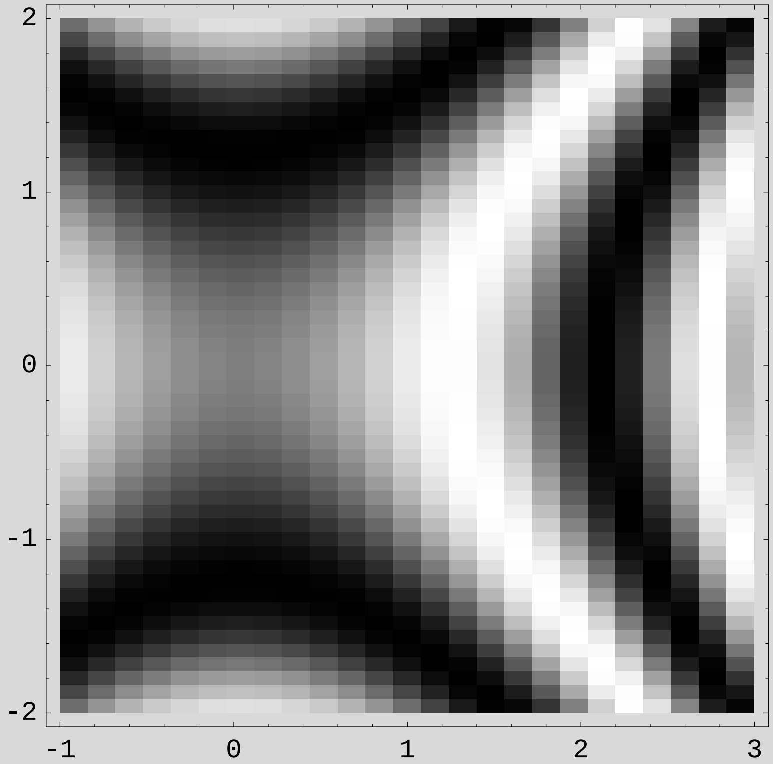
ΌpwV blέpoume crhsimopoioύntai 15 PlotPoints se kάqe έna apo ta diastήmata twn x kai y antίstoica. Gia megalύterh akrίbeia sta crώmata mporoύme na bάloume megalύterh timή p.c PlotPoints->{25,50}

```
gr1 = DensityPlot@Sin@x^2 − y^2D,
   8x, −1, 3<, 8y, −2, 2<, PlotPoints → 825, 50<D;
```



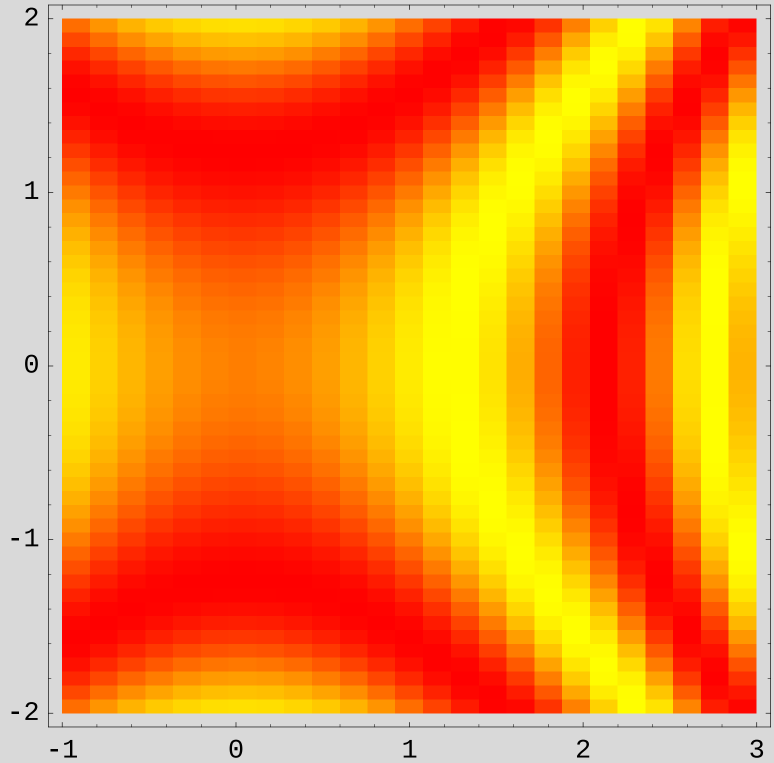
ΌpwV blέpoume den gínetai kammía prospáqeia na schmatistoún kapoia isouyή epípeda AplώV se káqe deigmatolhptikó shmeío apo ta 25 ä50 upologízetai h antístoich timή thV f kai sthn sunέceia autή metatrέpetai se mia apócrwsh tou gkrí. Me Mesh->False mporoúme na exajanísoume to plέgma kai na meínei móno h apócrwsh:

```
gr2 = Show@gr1, Mesh → FalseD;
```



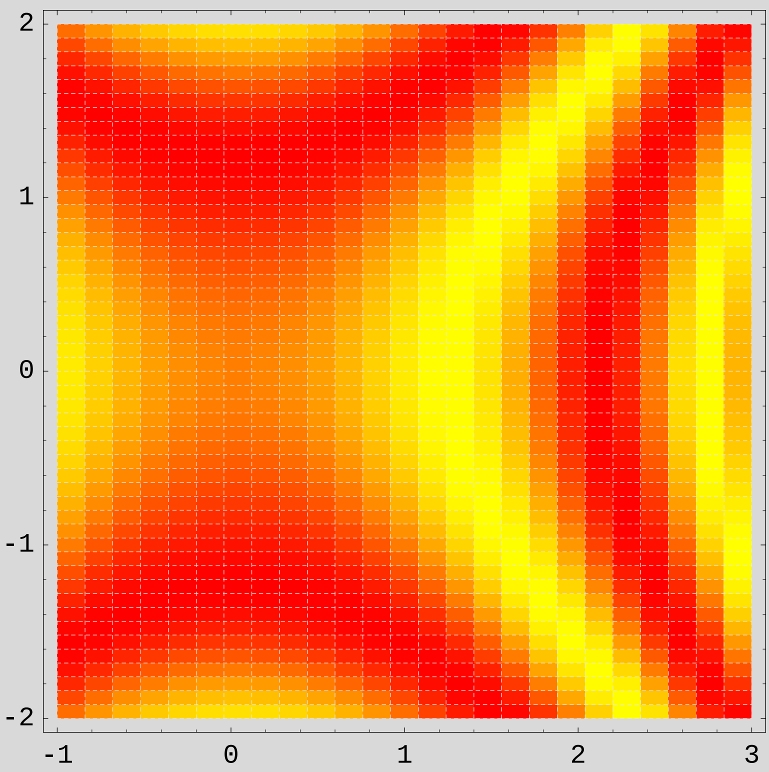
Me thn ColorFunction mporoúme na allάxoume katά boúlhsh tiV apocrώseiV:

```
gr3 = Show@gr2, ColorFunction → HRGBColor@1, #, 0D &LD;
```



An qέloume na emjanízontai oposdήpote kai to plέgma qa ήtan skópimo na dialέgame me thn boήqeia thV MeshStyle éna diajoretikó crώma grammώn plέgmatoV ή pio leptέV grammέV plέgmatoV ή diakekommέneV ή kάpoia apo ta prohgoúmena:

```
gr4 =
  Show@gr1, MeshStyle → 8GrayLevel@0.9D, Dashing@80.005, 0.005<D<,
   ColorFunction → HRGBColor@1, #, 0D &LD;
```

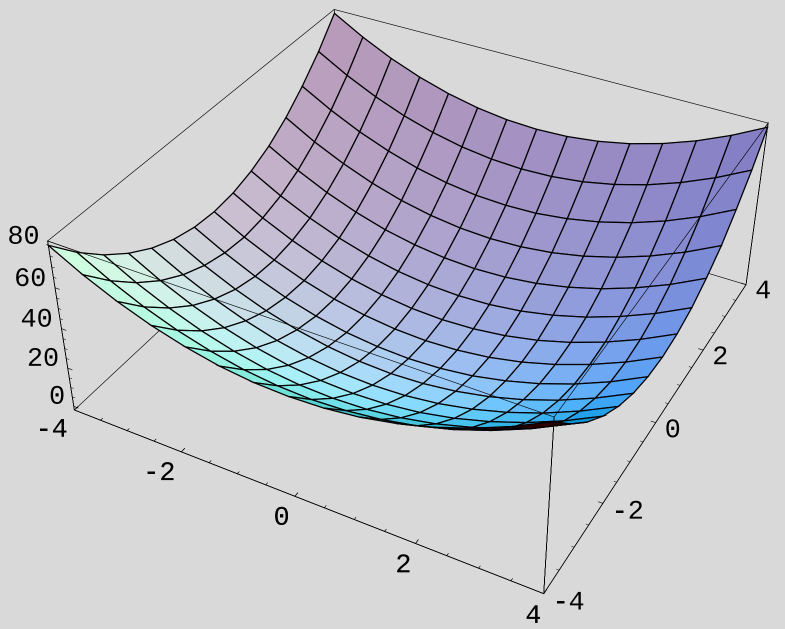


Genikά h DensityPlot maV scediάzei mia epijάneia tou cώrou όpwV qa thn έblepe έnaV parathrhtήV pou briskόtan akribώV apo pάnw thV!

ΊswV qa anarwtiέste giatί na crhsimopoiήsoume thn DensityPlot ajoύ upάrcei h ContourPlot. H apάnthsh eίnai όti upάrcoun kakέV periptώseiV pou h ContourPlot sthn prospάqeia thV na zwgrajίsei ta isouyή epίpeda den bgάzei kάpoia kάpoio κατανοητό grάjhma dhl. maV epistrέjei anakribέV grάjhma. Genikά qa prέpei na eίmaste se qέsh na paίrnoume όleV tiV plhrojorίeV pou maV creiάzontαι sthn melέth maV kάnontaV katάllhlo sunduasmό όlwn twn dunatotήtwn p.c

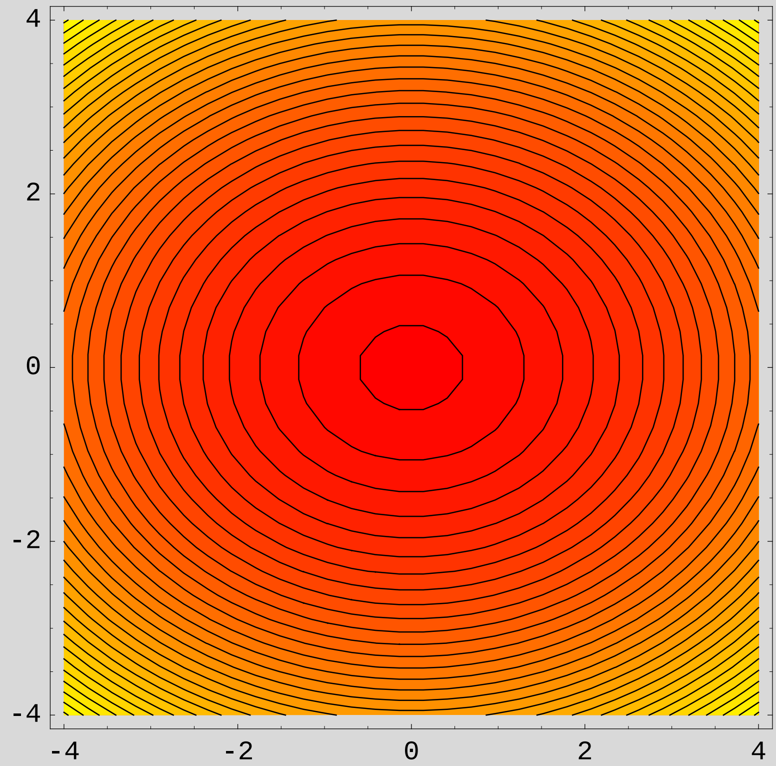
Askhsh: Na meletήsoume thn sumperijorά thV 2 x^2 + 3 y^2 gia {x,-4,4} kai {y,-4,4}.
Lύsh:Crhsimopoioύme thn Plot3D se sunduasmό me thn ContourPlot:

```
Plot3D@2 x^2 + 3 y^2 , 8x, −4, 4< , 8y, −4, 4<D;
```



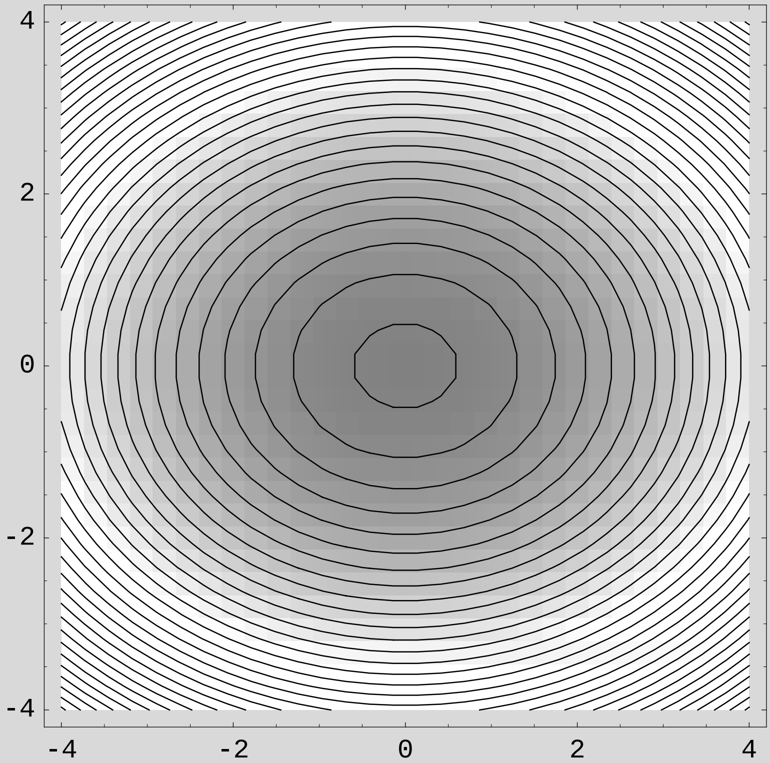
Blέpoume όti upάrcei έna baqoύlwma allά den xέroume pou akribώV. H ContourPlot qa bohqήsei ston entopismό tou:

```
ContourPlot@2 x^2 + 3 y^2 , 8x, −4, 4< ,
  8y, −4, 4<, ColorFunction → HRGBColor@1, #, 0D &L,
  Contours → 30, PlotPoints → 30D;
```



Dhl. **eínai to shmeío** (0,0)! **Qa mporoúsame na crhsimopoíhsoume kai thn** ContourPlot **se sunduasmó me thn** DensityPlot **wV exήV**: Θέτουμε την DensityPlot κάτω απο την ContourPlot και στην πρώτη βάζουμε MeshØFalse ενώ στην δεύτερη ContourShadingØFalse(για να εμφανιστούν μόνο οι ισουψείς καμπύλες)

```
cg = ContourPlot@2 x^2 + 3 y^2 ,
   8x, −4, 4< , 8y, −4, 4<, ContourShading → False,
   DisplayFunction → Identity, PlotPoints → 30, Contours → 30D;
dg = DensityPlot@2 x^2 + 3 y^2 , 8x, −4, 4< , 8y, −4, 4<,
   PlotPoints → 30, Mesh → False, DisplayFunction → Identity,
   ColorFunction → HGrayLevel@If@0.5 + # > 1, 1, 0.5 + #DD &LD;
Show@dg, cg, DisplayFunction → $DisplayFunctionD;
```



Me If[0.5+#>1,1,0.5+#] **jwtísame katá** 0.5 **perissótero ta skoteiná merh tou** DensityPlot **gia na écoume perissóterh jwteinóthta sta shmeía gúrw apo to** (0,0).