Thresholds for discrete solitons in the DNLS equation

We consider the DNLS equation in one dimension:

\[
\dot{\psi}_n = (\Delta \psi_n - n^2 \psi_n + U \psi_{n+1} + U \psi_{n-1} + V_n) \psi_n
\]

with \(U > 0\) a dimensionless index, \(\Delta\) the positive and the sign of \(\epsilon\) depends on the parity (attractive or repulsive) of the BEC. For an attractive BEC, \(U = 0\); for a repulsive BEC, \(U > 0\). Baryonic static states are given by:

\[
|\psi_0|, \psi_1, \psi_{-1}, |\psi_1|, |\psi_{-1}|
\]

In other words, the soliton wave is a threshold. In (Cuevas and Eilbeck 2006), finite-threshold states were obtained using a fixed point argument. For \(U > 0\) (repulsive BEC):

\[
|\psi_1|, |\psi_{-1}|
\]

This result is independent of the dimension of the lattice and the coupling constant \(U\). The following figure compares the thresholds analytically calculated with the real solitons.

Thresholds for discrete solitons in the cubic-quintic (CQ) DNLS equation

The dynamics of an attractive BEC loaded on an optical lattice can be approximately described by a CQ-DNLS equation if repulsive three-body interactions are taken into account (Michinel et al. 2002). The CQ-DNLS can be written as:

\[
\dot{\psi}_n = (\Delta \psi_n - n^2 \psi_n + U \psi_{n+1} + U \psi_{n-1} + V_n) \psi_n + \epsilon \psi_{n+1} f(\psi_n) + \epsilon \psi_{n-1} f(\psi_n)
\]

where \(f(\psi) = 25 - 3\psi^2 + \psi^4\). For \(U > 0\) (repulsive BEC):

\[
|\psi_1|, |\psi_{-1}|
\]

Thresholds for discrete solitons in the PR-DNLS equation

The following figure shows the \((U, K)\) parameter space together with the number of roots:

[Figure showing the parameter space for the PR-DNLS with \(U > 0\) and \(\epsilon > 0\). The graph illustrates the number of solitonic states for various values of \(U\) and \(K\).]

Comparison between the thresholds in the cubic DNLS, CQ-DNLS and PR-DNLS

Thresholds (red line) and quadratic norm (blue line) for solitons in the cubic DNLS (green) and PR-DNLS (red).

Radiationless travelling waves in the PR-DNLS equation

Contrary to continuous systems, which are translationally invariant, moving discrete solitons are strongly affected by the lattice. In particular, they obey discrete linear waves with radiation. This radiation is due to the minimum of a Fourier Transform (FT) potential (or barrier) whose origin is in the denominator of the lattice and whose significance is the bandwidth of the increment and, if this potential is deep enough, can make the intruder impossible. The PR-DNLS has been extensively studied to see that contact inelasticities are not described by the nonlinear term of the CQ-DNLS. The dynamics are affected by the lattice and its radiation mechanism used. This reaction is very different in the case of the CQ-DNLS where the potential barrier acts as a strong impenetrable wall except at small norms. This is illustrated in the following figure.

Moving discrete solitons in the PR-DNLS equation

The scenario of binary collisions of two identical moving discrete solitons in the CQ-DNLS equation that was studied by Cavaliere (2006) extends to the CQ-DNLS equation (Cavaliere et al. 2006) with the nonlinear potentials. The only way to obtain a radiationless travelling wave is that only one linear wave must be embedded. This linear wave is forced as a result of the lattice and the mechanism used. This reaction is very different in the case of the CQ-DNLS where the potential barrier acts as a strong impenetrable wall except at small norms. This is illustrated in the following figure.

Figure showing the dynamics of binary collisions in the PR-DNLS equation.

Figure showing the dynamics of binary collisions in the CQ-DNLS equation.

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References

[List of references related to the topics discussed in the poster.]